

Workshop Solutions to Sections 5.1 and 5.2

<p>1) The absolute maximum value of $f(x) = x^3 - 2x^2$ in $[-1, 2]$ is at $x =$</p> <p><u>Solution:</u> Since $f(x)$ is a continuous on $[-1, 2]$, we can use the Closed Interval Method,</p> $f(x) = x^3 - 2x^2$ $f'(x) = 3x^2 - 4x$ <p>Now, we find the critical numbers of $f(x)$ when</p> $f'(x) = 0 \Rightarrow 3x^2 - 4x = 0 \Rightarrow x(3x - 4) = 0$ $\Rightarrow x = 0 \text{ or } x = \frac{4}{3}$ <p>Thus,</p> $f(-1) = (-1)^3 - 2(-1)^2 = -1 - 2 = -3$ $f(2) = (2)^3 - 2(2)^2 = 8 - 8 = 0$ $f(0) = (0)^3 - 2(0)^2 = 0 - 0 = 0$ $f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2 = \frac{64}{27} - \frac{32}{9} = -\frac{32}{27}$ <p>Hence, we see that the absolute maximum value is 0 at $x = 0$ and $x = 2$</p>	<p>2) The absolute minimum value of $f(x) = x^3 - 3x^2 + 1$ in $\left[-\frac{1}{2}, 4\right]$ is</p> <p><u>Solution:</u> Since $f(x)$ is a continuous on $\left[-\frac{1}{2}, 4\right]$, we can use the Closed Interval Method,</p> $f(x) = x^3 - 3x^2 + 1$ $f'(x) = 3x^2 - 6x$ <p>Now, we find the critical numbers of $f(x)$ when</p> $f'(x) = 0 \Rightarrow 3x^2 - 6x = 0 \Rightarrow 3x(x - 2) = 0$ $\Rightarrow x = 0 \text{ or } x = 2$ <p>Thus,</p> $f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 1 = -\frac{1}{8} - \frac{3}{4} + 1 = \frac{1}{8}$ $f(4) = (4)^3 - 3(4)^2 + 1 = 64 - 48 + 1 = 17$ $f(0) = (0)^3 - 3(0)^2 + 1 = 0 - 0 + 1 = 1$ $f(2) = (2)^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3$ <p>Hence, we see that the absolute minimum value is -3 at $x = 2$</p>
<p>3) The absolute maximum point of $f(x) = 3x^2 - 12x + 1$ in $[0, 3]$ is</p> <p><u>Solution:</u> Since $f(x)$ is a continuous on $[0, 3]$, we can use the Closed Interval Method,</p> $f(x) = 3x^2 - 12x + 1$ $f'(x) = 6x - 12$ <p>Now, we find the critical numbers of $f(x)$ when</p> $f'(x) = 0 \Rightarrow 6x - 12 = 0 \Rightarrow 6x = 12$ $\Rightarrow x = 2$ <p>Thus,</p> $f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$ $f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$ $f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$ <p>Hence, we see that the absolute maximum point is $(0, 1)$.</p>	<p>4) The absolute minimum point of $f(x) = 3x^2 - 12x + 1$ in $[0, 3]$ is</p> <p><u>Solution:</u> Since $f(x)$ is a continuous on $[0, 3]$, we can use the Closed Interval Method,</p> $f(x) = 3x^2 - 12x + 1$ $f'(x) = 6x - 12$ <p>Now, we find the critical numbers of $f(x)$ when</p> $f'(x) = 0 \Rightarrow 6x - 12 = 0 \Rightarrow 6x = 12$ $\Rightarrow x = 2$ <p>Thus,</p> $f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$ $f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$ $f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$ <p>Hence, we see that the absolute minimum point is $(2, -11)$.</p>
<p>5) The absolute minimum point of $f(x) = 3x^2 - 12x + 2$ in $[0, 3]$ is</p> <p><u>Solution:</u> Since $f(x)$ is a continuous on $[0, 3]$, we can use the Closed Interval Method,</p> $f(x) = 3x^2 - 12x + 2$ $f'(x) = 6x - 12$ <p>Now, we find the critical numbers of $f(x)$ when</p> $f'(x) = 0 \Rightarrow 6x - 12 = 0 \Rightarrow 6x = 12$ $\Rightarrow x = 2$ <p>Thus,</p> $f(0) = 3(0)^2 - 12(0) + 2 = 0 - 0 + 2 = 2$ $f(3) = 3(3)^2 - 12(3) + 2 = 27 - 36 + 2 = -7$ $f(2) = 3(2)^2 - 12(2) + 2 = 12 - 24 + 2 = -10$ <p>Hence, we see that the absolute minimum point is $(2, -10)$.</p>	<p>6) The values in $(-3, 3)$ which make $f(x) = x^3 - 9x$ satisfy Rolle's Theorem on $[-3, 3]$ are</p> <p><u>Solution:</u> $\because f(x)$ is a polynomial, then</p> <ol style="list-style-type: none"> 1- $f(x)$ is a continuous on $[-3, 3]$. 2- $f(x)$ is differentiable on $(-3, 3)$, $f'(x) = 3x^2 - 9$ 3- $f(-3) = (-3)^3 - 9(-3) = -27 + 27 = 0 = f(3)$ <p>Then there is a number $c \in (-3, 3)$ such that</p> $f'(c) = 0 \Rightarrow 3c^2 - 9 = 0 \Rightarrow 3c^2 = 9$ $\Rightarrow c^2 = 3 \Rightarrow c = \pm\sqrt{3}$ <p>Hence, the values are $\pm\sqrt{3} \in (-3, 3)$.</p>

7) The values in $(0,2)$ which make $f(x) = x^3 - 3x^2 + 2x + 5$ satisfy Rolle's Theorem on $[0,2]$ are

Solution:

$\because f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,2]$.

2- $f(x)$ is differentiable on $(0,2)$,

$$f'(x) = 3x^2 - 6x + 2$$

3- $f(0) = (0)^3 - 3(0)^2 + 2(0) + 5 = 5 = f(2)$

Then there is a number $c \in (0,2)$ such that

$$f'(c) = 0 \Rightarrow 3c^2 - 6c + 2 = 0$$

$$\begin{aligned} \Rightarrow c &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)} = \frac{6 \pm \sqrt{36 - 24}}{6} \\ &= \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm \sqrt{3} \times 2}{6} = \frac{6 \pm 2\sqrt{3}}{6} \\ &= \frac{2(3 \pm \sqrt{3})}{6} = \frac{3 \pm \sqrt{3}}{3} = \frac{3}{3} \pm \frac{\sqrt{3}}{3} \\ &= 1 \pm \frac{\sqrt{3}}{3} \end{aligned}$$

Hence, the values are $1 \pm \frac{\sqrt{3}}{3} \in (0,2)$.

8) The value c in $(0,5)$ which makes $f(x) = x^2 - x - 6$ satisfy the Mean Value Theorem on $[0,5]$ is

Solution:

$\because f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,5]$.

2- $f(x)$ is differentiable on $(0,5)$,

$$f'(x) = 2x - 1$$

Then there is a number $c \in (0,5)$ such that

$$\begin{aligned} f'(c) &= \frac{f(5) - f(0)}{5 - 0} \\ \Rightarrow 2c - 1 &= \frac{[(5)^2 - (5) - 6] - [(0)^2 - (0) - 6]}{5} \\ \Rightarrow 2c - 1 &= \frac{(14) - (-6)}{5} \\ \Rightarrow 2c - 1 &= \frac{14 + 6}{5} \\ \Rightarrow 2c - 1 &= 4 \\ \Rightarrow 2c &= 4 + 1 \\ \Rightarrow c &= \frac{5}{2} \end{aligned}$$

Hence, the value c is $\frac{5}{2} \in (0,5)$.

9) The value c in $(0,2)$ makes $f(x) = x^3 - x$ satisfied the Mean Value Theorem on $[0,2]$ are

Solution:

$\because f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,2]$.

2- $f(x)$ is differentiable on $(0,2)$,

$$f'(x) = 3x^2 - 1$$

Then there is a number $c \in (0,2)$ such that

$$\begin{aligned} f'(c) &= \frac{f(2) - f(0)}{2 - 0} \\ \Rightarrow 3c^2 - 1 &= \frac{[(2)^3 - (2)] - [(0)^3 - (0)]}{2} \\ \Rightarrow 3c^2 - 1 &= \frac{(6) - (0)}{2} \\ \Rightarrow 3c^2 - 1 &= \frac{6}{2} \\ \Rightarrow 3c^2 - 1 &= 3 \\ \Rightarrow 3c^2 &= 3 + 1 \\ \Rightarrow c^2 &= \frac{4}{3} \\ \Rightarrow c &= \pm \sqrt{\frac{4}{3}} \\ \Rightarrow c &= \pm \frac{2}{\sqrt{3}} \end{aligned}$$

Hence, the value c is $\frac{2}{\sqrt{3}} \in (0,2)$ but $-\frac{2}{\sqrt{3}} \notin (0,2)$.

10) The value in $(0,1)$ which makes $f(x) = 3x^2 + 2x + 5$ satisfy the Mean Value Theorem on $[0,1]$ is

Solution:

$\because f(x)$ is a polynomial, then

1- $f(x)$ is a continuous on $[0,1]$.

2- $f(x)$ is differentiable on $(0,1)$,

$$f'(x) = 6x + 2$$

Then there is a number $c \in (0,1)$ such that

$$\begin{aligned} f'(c) &= \frac{f(1) - f(0)}{1 - 0} \\ \Rightarrow 6c + 2 &= \frac{[3(1)^2 + 2(1) + 5] - [3(0)^2 + 2(0) + 5]}{1} \\ \Rightarrow 6c + 2 &= (3 + 2 + 5) - (0 + 0 + 5) \\ \Rightarrow 6c + 2 &= 10 - 5 \\ \Rightarrow 6c + 2 &= 5 \\ \Rightarrow 6c &= 5 - 2 \\ \Rightarrow 6c &= 3 \\ \Rightarrow c &= \frac{3}{6} \\ \Rightarrow c &= \frac{1}{2} \end{aligned}$$

Hence, the values are $\frac{1}{2} \in (0,1)$.

11) The critical numbers of the function

$$f(x) = x^3 + 3x^2 - 9x + 1 \text{ are}$$

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \end{aligned}$$

12) The function $f(x) = x^3 + 3x^2 - 9x + 1$ is decreasing on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \\ &\quad -3 \qquad \qquad 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity
			y

Hence, the function $f(x)$ is decreasing on $(-3, 1)$

13) The function $f(x) = x^3 + 3x^2 - 9x + 1$ is increasing on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \\ &\quad -3 \qquad \qquad 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity
			y

Hence, the function $f(x)$ is increasing on $(-\infty, -3) \cup (1, \infty)$

14) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has a relative maximum value at the point

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \\ &\quad -3 \qquad \qquad 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity
			y

Hence, the function $f(x)$ has a relative maximum value at the point $(-3, 28)$.

$$\begin{aligned} f(-3) &= (-3)^3 + 3(-3)^2 - 9(-3) + 1 \\ &= -27 + 27 + 27 + 1 = 28 \end{aligned}$$

15) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has a relative minimum value at the point

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \\ &\quad -3 \qquad \qquad 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity
			y

Hence, the function $f(x)$ has a relative minimum value at the point $(1, -4)$.

$$\begin{aligned} f(1) &= (1)^3 + 3(1)^2 - 9(1) + 1 \\ &= 1 + 3 - 9 + 1 = -4 \end{aligned}$$

16) The function $f(x) = x^3 + 3x^2 - 9x + 1$ concave upward on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f''(x) &= 6x + 6 \\ f''(x) = 0 &\Rightarrow 6x + 6 = 0 \\ &\Rightarrow 6x = -6 \\ &\Rightarrow x = -\frac{6}{6} \\ &\Rightarrow x = -1 \\ &\quad -1 \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave upward on $(-1, \infty)$

17) The function $f(x) = x^3 + 3x^2 - 9x + 1$ concave downward on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f''(x) &= 6x + 6 \\ f''(x) = 0 &\Rightarrow 6x + 6 = 0 \\ &\Rightarrow 6x = -6 \\ &\Rightarrow x = -\frac{6}{6} \\ &\Rightarrow x = -1 \\ &\quad -1 \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, -1)$

18) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has an inflection point at

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$$f''(x) = 0 \Rightarrow 6x + 6 = 0$$

$$\Rightarrow 6x = -6$$

$$\Rightarrow x = -\frac{6}{6}$$

$$\Rightarrow x = -1$$

$$-1$$

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(-1, 12)$.

$$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) + 1$$

$$= -1 + 3 + 9 + 1 = 12$$

19) The critical numbers of the function $f(x) = x^3 - 3x^2 - 9x + 1$ are

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

20) The function $f(x) = x^3 - 3x^2 - 9x + 1$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

$$-1$$

$$3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-1, 3)$

21) The function $f(x) = x^3 - 3x^2 - 9x + 1$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

$$-1$$

$$3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -1) \cup (3, \infty)$

22) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has a relative maximum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

$$-1$$

$$3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-1, 6)$.

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 1$$

$$= -1 - 3 + 9 + 1 = 6.$$

23) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has a relative minimum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

$$-1$$

$$3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(3, -26)$.

$$f(3) = (3)^3 - 3(3)^2 - 9(3) + 1$$

$$= 27 - 27 - 27 + 1 = -26.$$

24) The function $f(x) = x^3 - 3x^2 - 9x + 1$ concave upward on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

1

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$

25) The function $f(x) = x^3 - 3x^2 - 9x + 1$ concave downward on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

1

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$

26) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

1

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1, -10)$.

$$f(1) = (1)^3 - 3(1)^2 - 9(1) + 1$$

$$= 1 - 3 - 9 + 1 = -10$$

27) The critical numbers of the function $f(x) = x^3 + 3x^2 - 9x + 5$ are

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$

$$\Rightarrow 3(x^2 + 2x - 3) = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x + 3)(x - 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 1$$

28) The function $f(x) = x^3 + 3x^2 - 9x + 5$ is decreasing on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$

$$\Rightarrow 3(x^2 + 2x - 3) = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x + 3)(x - 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 1$$

-3

1

+	-	+	Sign of $f'(x)$
\nearrow	\searrow	\nearrow	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-3, 1)$.

29) The function $f(x) = x^3 + 3x^2 - 9x + 5$ is increasing on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$

$$\Rightarrow 3(x^2 + 2x - 3) = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x + 3)(x - 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 1$$

-3

1

+	-	+	Sign of $f'(x)$
\nearrow	\searrow	\nearrow	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -3) \cup (1, \infty)$.

30) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has a relative minimum value at the point

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$

$$\Rightarrow 3(x^2 + 2x - 3) = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x + 3)(x - 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 1$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity y

Hence, the function $f(x)$ has a relative minimum value at the point (1,0).

$$f(1) = (1)^3 + 3(1)^2 - 9(1) + 5$$

$$= 1 + 3 - 9 + 5 = 0$$

32) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has an inflection point at

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$$f''(x) = 0 \Rightarrow 6x + 6 = 0$$

$$\Rightarrow 6x = -6$$

$$\Rightarrow x = -\frac{6}{6}$$

$$\Rightarrow x = -1$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ has an inflection point at (-1,16).

$$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) + 5$$

$$= -1 + 3 + 9 + 5 = 16$$

34) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave upward on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$$f''(x) = 0 \Rightarrow 6x + 6 = 0$$

$$\Rightarrow 6x = -6$$

$$\Rightarrow x = -\frac{6}{6}$$

$$\Rightarrow x = -1$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave upward on $(-1, \infty)$.

31) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has a relative maximum value at the point

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

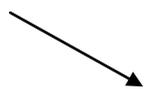
$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$

$$\Rightarrow 3(x^2 + 2x - 3) = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x + 3)(x - 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 1$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity y

Hence, the function $f(x)$ has a relative maximum value at the point (-3,32).

$$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 5$$

$$= -27 + 27 + 27 + 5 = 32$$

33) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave downward on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$$f''(x) = 0 \Rightarrow 6x + 6 = 0$$

$$\Rightarrow 6x = -6$$

$$\Rightarrow x = -\frac{6}{6}$$

$$\Rightarrow x = -1$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, -1)$.

35) The critical numbers of the function $f(x) = x^3 - 3x^2 - 9x + 5$ are

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

36) The function $f(x) = x^3 - 3x^2 - 9x + 5$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -1) \cup (3, \infty)$.

37) The function $f(x) = x^3 - 3x^2 - 9x + 5$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-1, 3)$.

38) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative maximum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-1, 10)$.

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 5$$

$$= -1 - 3 + 9 + 5 = 10.$$

39) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative minimum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(3, -22)$.

$$f(3) = (3)^3 - 3(3)^2 - 9(3) + 5$$

$$= 27 - 27 - 27 + 5 = -22.$$

40) The function $f(x) = x^3 - 3x^2 - 9x + 5$ concave upward on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$.

41) The function $f(x) = x^3 - 3x^2 - 9x + 5$ concave downward on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.

42) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1, -6)$.

$$f(1) = (1)^3 - 3(1)^2 - 9(1) + 5$$

$$= 1 - 3 - 9 + 5 = -6$$

43) The critical numbers of the function

$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$$
 are

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

44) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ is increasing on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

$$\begin{matrix} -1 & & 2 \end{matrix}$$

+	-	+	Sign of $f'(x)$
\nearrow	\searrow	\nearrow	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -1) \cup (2, \infty)$.

45) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ is decreasing on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

$$\begin{matrix} -1 & & 2 \end{matrix}$$

+	-	+	Sign of $f'(x)$
\nearrow	\searrow	\nearrow	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-1, 2)$.

46) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ has a relative maximum point

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

$$\begin{matrix} -1 & & 2 \end{matrix}$$

+	-	+	Sign of $f'(x)$
\nearrow	\searrow	\nearrow	Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $(-1, \frac{13}{6})$.

$$f(-1) = \frac{1}{3}(-1)^3 - \frac{1}{2}(-1)^2 - 2(-1) + 1$$

$$= -\frac{1}{3} - \frac{1}{2} + 2 + 1 = \frac{13}{6}$$

47) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ has a relative minimum point

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

$$\begin{matrix} -1 & & 2 \end{matrix}$$

+	-	+	Sign of $f'(x)$
\nearrow	\searrow	\nearrow	Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $(2, -\frac{7}{3})$.

$$f(2) = \frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 - 2(2) + 1$$

$$= \frac{8}{3} - \frac{4}{2} - 4 + 1 = -\frac{7}{3}$$

48) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ concave upward on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f''(x) = 2x - 1$$

$$f''(x) = 0 \Rightarrow 2x - 1 = 0$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$\frac{1}{2}$

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ is concave upward on $\left(\frac{1}{2}, \infty\right)$.

49) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ concave downward on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f''(x) = 2x - 1$$

$$f''(x) = 0 \Rightarrow 2x - 1 = 0$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$\frac{1}{2}$

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ is concave downward on $\left(-\infty, \frac{1}{2}\right)$.

50) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ has an inflection point at

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f''(x) = 2x - 1$$

$$f''(x) = 0 \Rightarrow 2x - 1 = 0$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$\frac{1}{2}$

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ has an inflection point at

$\left(\frac{1}{2}, -\frac{1}{12}\right)$.

$$f\left(\frac{1}{2}\right) = \frac{1}{3}\left(\frac{1}{2}\right)^3 - \frac{1}{2}\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{24} - \frac{1}{8} - 1 + 1 = -\frac{1}{12}$$

51) The critical numbers of the function

$$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$$
 are

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$f'(x) = 0 \Rightarrow x^2 + x - 2 = 0$$

$$\Rightarrow (x+2)(x-1) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

52) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ is increasing on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$f'(x) = 0 \Rightarrow x^2 + x - 2 = 0$$

$$\Rightarrow (x+2)(x-1) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

-2 1

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -2) \cup (1, \infty)$.

53) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ is decreasing on

Solution:

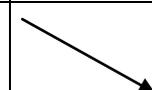
$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$f'(x) = 0 \Rightarrow x^2 + x - 2 = 0$$

$$\Rightarrow (x+2)(x-1) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

-2 1

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

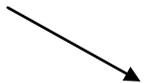
Hence, the function $f(x)$ is decreasing on $(-2, 1)$.

54) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ has a relative maximum point

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$\begin{aligned} f'(x) = 0 &\Rightarrow x^2 + x - 2 = 0 \\ &\Rightarrow (x+2)(x-1) = 0 \\ &\Rightarrow x = -2 \text{ or } x = 1 \\ &\quad -2 \qquad \qquad 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $\left(-2, \frac{13}{3}\right)$.

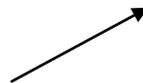
$$\begin{aligned} f(-2) &= \frac{1}{3}(-2)^3 + \frac{1}{2}(-2)^2 - 2(-2) + 1 \\ &= -\frac{8}{3} + \frac{4}{2} + 4 + 1 = \frac{13}{3} \end{aligned}$$

55) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ has a relative minimum point

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$\begin{aligned} f'(x) = 0 &\Rightarrow x^2 + x - 2 = 0 \\ &\Rightarrow (x+2)(x-1) = 0 \\ &\Rightarrow x = -2 \text{ or } x = 1 \\ &\quad -2 \qquad \qquad 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $\left(1, -\frac{1}{6}\right)$.

$$\begin{aligned} f(1) &= \frac{1}{3}(1)^3 + \frac{1}{2}(1)^2 - 2(1) + 1 \\ &= \frac{1}{3} + \frac{1}{2} - 2 + 1 = -\frac{1}{6} \end{aligned}$$

56) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ concave upward on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$f''(x) = 2x + 1$$

$$\begin{aligned} f''(x) = 0 &\Rightarrow 2x + 1 = 0 \\ &\Rightarrow 2x = -1 \\ &\Rightarrow x = -\frac{1}{2} \\ &\quad -\frac{1}{2} \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave upward on $\left(-\frac{1}{2}, \infty\right)$.

57) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ concave downward on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$f''(x) = 2x + 1$$

$$\begin{aligned} f''(x) = 0 &\Rightarrow 2x + 1 = 0 \\ &\Rightarrow 2x = -1 \\ &\Rightarrow x = -\frac{1}{2} \\ &\quad -\frac{1}{2} \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave downward on $\left(-\infty, -\frac{1}{2}\right)$.

58) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ has an inflection point at

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$f''(x) = 2x + 1$$

$$\begin{aligned} f''(x) = 0 &\Rightarrow 2x + 1 = 0 \\ &\Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2} \\ &\quad -\frac{1}{2} \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ has an inflection point at $\left(-\frac{1}{2}, \frac{25}{12}\right)$.

$$\begin{aligned} f\left(-\frac{1}{2}\right) &= \frac{1}{3}\left(-\frac{1}{2}\right)^3 + \frac{1}{2}\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1 \\ &= -\frac{1}{24} + \frac{1}{8} + 1 + 1 = \frac{25}{12} \end{aligned}$$

59) The critical numbers of the function $f(x) = x^3 - 12x + 3$ are

Solution:

$$f'(x) = 3x^2 - 12$$

$$\begin{aligned} f'(x) = 0 &\Rightarrow 3x^2 - 12 = 0 \\ &\Rightarrow 3(x^2 - 4) = 0 \\ &\Rightarrow x^2 - 4 = 0 \\ &\Rightarrow x^2 = 4 \\ &\Rightarrow x = \pm 2 \end{aligned}$$

60) The function $f(x) = x^3 - 12x + 3$ is increasing on

Solution:

$$f'(x) = 3x^2 - 12$$

$$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0$$

$$\Rightarrow 3(x^2 - 4) = 0$$

$$\Rightarrow x^2 - 4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -2) \cup (2, \infty)$.

61) The function $f(x) = x^3 - 12x + 3$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 12$$

$$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0$$

$$\Rightarrow 3(x^2 - 4) = 0$$

$$\Rightarrow x^2 - 4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-2, 2)$.

62) The function $f(x) = x^3 - 12x + 3$ has a relative maximum point at

Solution:

$$f'(x) = 3x^2 - 12$$

$$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0$$

$$\Rightarrow 3(x^2 - 4) = 0$$

$$\Rightarrow x^2 - 4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $(-2, 19)$.

$$f(-2) = (-2)^3 - 12(-2) + 3$$

$$= -8 + 24 + 3 = 19.$$

63) The function $f(x) = x^3 - 12x + 3$ has a relative minimum point at

Solution:

$$f'(x) = 3x^2 - 12$$

$$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0$$

$$\Rightarrow 3(x^2 - 4) = 0$$

$$\Rightarrow x^2 - 4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $(2, -13)$.

$$f(2) = (2)^3 - 12(2) + 3$$

$$= 8 - 24 + 3 = -13$$

64) The function $f(x) = x^3 - 12x + 3$ concave upward on

Solution:

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x$$

$$f''(x) = 0 \Rightarrow 6x = 0$$

$$\Rightarrow x = \frac{0}{6}$$

$$\Rightarrow x = 0$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave upward on $(0, \infty)$.

65) The function $f(x) = x^3 - 12x + 3$ concave downward on

Solution:

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x$$

$$f''(x) = 0 \Rightarrow 6x = 0$$

$$\Rightarrow x = \frac{0}{6}$$

$$\Rightarrow x = 0$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 0)$.

66) The function $f(x) = x^3 - 12x + 3$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x$$

$$f''(x) = 0 \Rightarrow 6x = 0$$

$$\Rightarrow x = \frac{0}{6}$$

$$\Rightarrow x = 0$$

0

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(0,3)$.

$$f(0) = (0)^3 - 12(0)^2 + 3$$

$$= 0 - 0 + 3 = 3$$

67) The critical numbers of the function $f(x) = x^3 - 3x^2 + 1$ are

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

68) The function $f(x) = x^3 - 3x^2 + 1$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

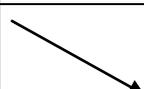
$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

0

2

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$.

69) The function $f(x) = x^3 - 3x^2 + 1$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

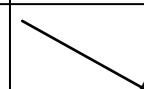
$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

0

2

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(0,2)$.

70) The function $f(x) = x^3 - 3x^2 + 1$ has a relative maximum point at

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

0

2

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $(0,1)$.

$$f(0) = (0)^3 - 3(0)^2 + 1$$

$$= 0 - 0 + 1 = 1.$$

71) The function $f(x) = x^3 - 3x^2 + 1$ has a relative minimum point at

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

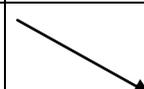
$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

0

2

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $(2,-3)$.

$$f(2) = (2)^3 - 3(2)^2 + 1$$

$$= 8 - 12 + 1 = -3.$$

72) The function $f(x) = x^3 - 3x^2 + 1$ concave upward on

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

1

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$.

73) The function $f(x) = x^3 - 3x^2 + 1$ concave downward on

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

1

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.

74) The function $f(x) = x^3 - 3x^2 + 1$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

1

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1, -1)$.

$$f(1) = (1)^3 - 3(1)^2 + 1$$

$$= 1 - 3 + 1 = -1$$

75) The critical numbers of the function $f(x) = x^3 - 3x^2 + 2$ are

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

76) The function $f(x) = x^3 - 3x^2 + 2$ is increasing on

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

0

2

+	-	+	Sign of $f'(x)$
\nearrow	\searrow	\nearrow	Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$.

77) The function $f(x) = x^3 - 3x^2 + 2$ is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

0

2

+	-	+	Sign of $f'(x)$
\nearrow	\searrow	\nearrow	Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(0, 2)$.

78) The function $f(x) = x^3 - 3x^2 + 2$ has a relative minimum point at

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum point at $(2, -2)$.

$$f(2) = (2)^3 - 3(2)^2 + 2$$

$$= 8 - 12 + 2 = -2.$$

79) The function $f(x) = x^3 - 3x^2 + 2$ has a relative maximum point at

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum point at $(0, 2)$.

$$f(0) = (0)^3 - 3(0)^2 + 2$$

$$= 0 - 0 + 2 = 2.$$

80) The function $f(x) = x^3 - 3x^2 + 2$ concave downward on

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$.

81) The function $f(x) = x^3 - 3x^2 + 2$ concave upward on

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave upward on $(1, \infty)$.

82) The function $f(x) = x^3 - 3x^2 + 2$ has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(1, 0)$.

$$f(1) = (1)^3 - 3(1)^2 + 2$$

$$= 1 - 3 + 2 = 0$$

83) The critical numbers of the function $f(x) = x^3 - 6x^2 - 36x$ are

Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f'(x) = 0 \Rightarrow 3x^2 - 12x - 36 = 0$$

$$\Rightarrow 3(x^2 - 4x - 12) = 0$$

$$\Rightarrow x^2 - 4x - 12 = 0$$

$$\Rightarrow (x + 2)(x - 6) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 6$$

84) The function $f(x) = x^3 - 6x^2 - 36x$ is decreasing on
Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f'(x) = 0 \Rightarrow 3x^2 - 12x - 36 = 0$$

$$\Rightarrow 3(x^2 - 4x - 12) = 0$$

$$\Rightarrow x^2 - 4x - 12 = 0$$

$$\Rightarrow (x + 2)(x - 6) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 6$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-2, 6)$.

85) The function $f(x) = x^3 - 6x^2 - 36x$ is increasing on
Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f'(x) = 0 \Rightarrow 3x^2 - 12x - 36 = 0$$

$$\Rightarrow 3(x^2 - 4x - 12) = 0$$

$$\Rightarrow x^2 - 4x - 12 = 0$$

$$\Rightarrow (x + 2)(x - 6) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 6$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-\infty, -2) \cup (6, \infty)$.

86) The function $f(x) = x^3 - 6x^2 - 36x$ has a relative minimum value at the point
Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f'(x) = 0 \Rightarrow 3x^2 - 12x - 36 = 0$$

$$\Rightarrow 3(x^2 - 4x - 12) = 0$$

$$\Rightarrow x^2 - 4x - 12 = 0$$

$$\Rightarrow (x + 2)(x - 6) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 6$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(6, -216)$.

$$f(6) = (6)^3 - 6(6)^2 - 36(6)$$

$$= 216 - 216 - 216 = -216$$

87) The function $f(x) = x^3 - 6x^2 - 36x$ has a relative maximum value at the point
Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f'(x) = 0 \Rightarrow 3x^2 - 12x - 36 = 0$$

$$\Rightarrow 3(x^2 - 4x - 12) = 0$$

$$\Rightarrow x^2 - 4x - 12 = 0$$

$$\Rightarrow (x + 2)(x - 6) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 6$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-2, 40)$.

$$f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$$

$$= -8 - 24 + 72 = 40$$

88) The function $f(x) = x^3 - 6x^2 - 36x$ has an inflection point at
Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f''(x) = 6x - 12$$

$$f''(x) = 0 \Rightarrow 6x - 12 = 0$$

$$\Rightarrow 6x = 12$$

$$\Rightarrow x = \frac{12}{6}$$

$$\Rightarrow x = 2$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(2, -88)$.

$$f(2) = (2)^3 - 6(2)^2 - 36(2)$$

$$= 8 - 24 - 72 = -88$$

89) The function $f(x) = x^3 - 6x^2 - 36x$ concave downward on
Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f''(x) = 6x - 12$$

$$f''(x) = 0 \Rightarrow 6x - 12 = 0$$

$$\Rightarrow 6x = 12$$

$$\Rightarrow x = \frac{12}{6}$$

$$\Rightarrow x = 2$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-\infty, 2)$.

90) The function $f(x) = x^3 - 6x^2 - 36x$ concave upward on

Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f''(x) = 6x - 12$$

$$f''(x) = 0 \Rightarrow 6x - 12 = 0$$

$$\Rightarrow 6x = 12$$

$$\Rightarrow x = \frac{12}{6}$$

$$\Rightarrow x = 2$$

2

-	+	Sign of $f''(x)$
\cap	\cup	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(2, \infty)$.

91) The critical numbers of the function $f(x) = -x^3 - 6x^2 - 9x + 1$ are

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$$

$$\Rightarrow -3(x^2 + 4x + 3) = 0$$

$$\Rightarrow x^2 + 4x + 3 = 0$$

$$\Rightarrow (x + 3)(x + 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = -1$$

92) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ is decreasing on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$$

$$\Rightarrow -3(x^2 + 4x + 3) = 0$$

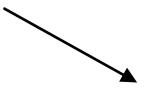
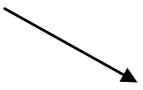
$$\Rightarrow x^2 + 4x + 3 = 0$$

$$\Rightarrow (x + 3)(x + 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = -1$$

-3

-1

-	+	-	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is decreasing on $(-\infty, -3) \cup (-1, \infty)$.

93) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ is increasing on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$$

$$\Rightarrow -3(x^2 + 4x + 3) = 0$$

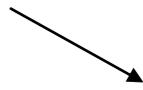
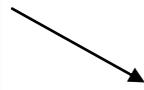
$$\Rightarrow x^2 + 4x + 3 = 0$$

$$\Rightarrow (x + 3)(x + 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = -1$$

-3

-1

-	+	-	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ is increasing on $(-3, -1)$.

94) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has a relative minimum value at the point

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$$

$$\Rightarrow -3(x^2 + 4x + 3) = 0$$

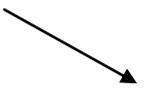
$$\Rightarrow x^2 + 4x + 3 = 0$$

$$\Rightarrow (x + 3)(x + 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = -1$$

-3

-1

-	+	-	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative minimum value at the point $(-3, 1)$.

$$f(-3) = -(-3)^3 - 6(-3)^2 - 9(-3) + 1$$

$$= 27 - 54 + 27 + 1 = 1.$$

95) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has a relative maximum value at the point

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$$

$$\Rightarrow -3(x^2 + 4x + 3) = 0$$

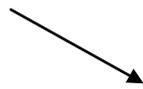
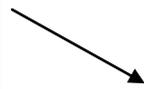
$$\Rightarrow x^2 + 4x + 3 = 0$$

$$\Rightarrow (x + 3)(x + 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = -1$$

-3

-1

-	+	-	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function $f(x)$ has a relative maximum value at the point $(-1, 5)$.

$$f(-1) = -(-1)^3 - 6(-1)^2 - 9(-1) + 1$$

$$= 1 - 6 + 9 + 1 = 5.$$

96) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has an inflection point at

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$$f''(x) = -6x - 12$$

$$f''(x) = 0 \Rightarrow -6x - 12 = 0$$

$$\Rightarrow -6x = 12$$

$$\Rightarrow x = -\frac{12}{6}$$

$$\Rightarrow x = -2$$

$$-2$$

+	-	Sign of $f''(x)$
U	∩	Kind of concavity

Hence, the function $f(x)$ has an inflection point at $(-2, 3)$.

$$f(-2) = -(-2)^3 - 6(-2)^2 - 9(-2) + 1$$

$$= 8 - 24 + 18 + 1 = 3$$

97) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ concave downward on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$$f''(x) = -6x - 12$$

$$f''(x) = 0 \Rightarrow -6x - 12 = 0$$

$$\Rightarrow -6x = 12$$

$$\Rightarrow x = -\frac{12}{6}$$

$$\Rightarrow x = -2$$

$$-2$$

+	-	Sign of $f''(x)$
U	∩	Kind of concavity

Hence, the function $f(x)$ is concave downward on $(-2, \infty)$.

98) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ concave upward on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$$f''(x) = -6x - 12$$

$$f''(x) = 0 \Rightarrow -6x - 12 = 0$$

$$\Rightarrow -6x = 12$$

$$\Rightarrow x = -\frac{12}{6}$$

$$\Rightarrow x = -2$$

$$-2$$

+	-	Sign of $f''(x)$
U	∩	Kind of concavity

Hence, the function $f(x)$ is concave upward on $(-\infty, -2)$.